technical proof

TP B.31
Percentage Sidespin Required for Maximum CIT at Any Cut Angle

supporting:
"The Illustrated Principles of Pool and Billiards"
https://drdavepoolinfo.com/
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From TP A.14, throw is calculated with the following, where speeds are in units of m/s:

$$R := \frac{1.125 \cdot in}{m} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} := \begin{pmatrix} 9.951 \times 10^{-3} \\ 0.108 \\ 1.088 \end{pmatrix} \qquad \mu(v) := a + b \cdot e^{-c \cdot v}$$

$$v_{rel}(v, \omega_x, \omega_z, \phi) := \sqrt{\left(v \cdot \sin(\phi) - R \cdot \omega_z\right)^2 + \left(R \cdot \omega_x \cdot \cos(\phi)\right)^2}$$

$$\theta_{throw}(v, \omega_x, \omega_z, \phi) := \operatorname{atan}\left[\frac{\min\left(\frac{\mu\left(v_{rel}(v, \omega_x, \omega_z, \phi)\right) \cdot v \cdot \cos(\phi)}{v_{rel}(v, \omega_x, \omega_z, \phi)}, \frac{1}{7}\right) \cdot \left(v \cdot \sin(\phi) - R \cdot \omega_z\right)}{v \cdot \cos(\phi)}\right]$$

From **https://drdavepoolinfo.com/faq/speed/typical/,** a typical range of CB speeds, converted to m/s is:

$$v_{slow} := \frac{1 \cdot mph}{\frac{m}{s}} = 0.447 \qquad v_{medium} := \frac{3 \cdot mph}{\frac{m}{s}} = 1.341 \qquad v_{fast} := \frac{7 \cdot mph}{\frac{m}{s}} = 3.129$$

From TP A.25, percentage spin (PS) is related to spin rate ω (rad/sec) with:

$$\omega(v, PS) := \frac{5}{4} \cdot \frac{v}{R} \cdot PS$$

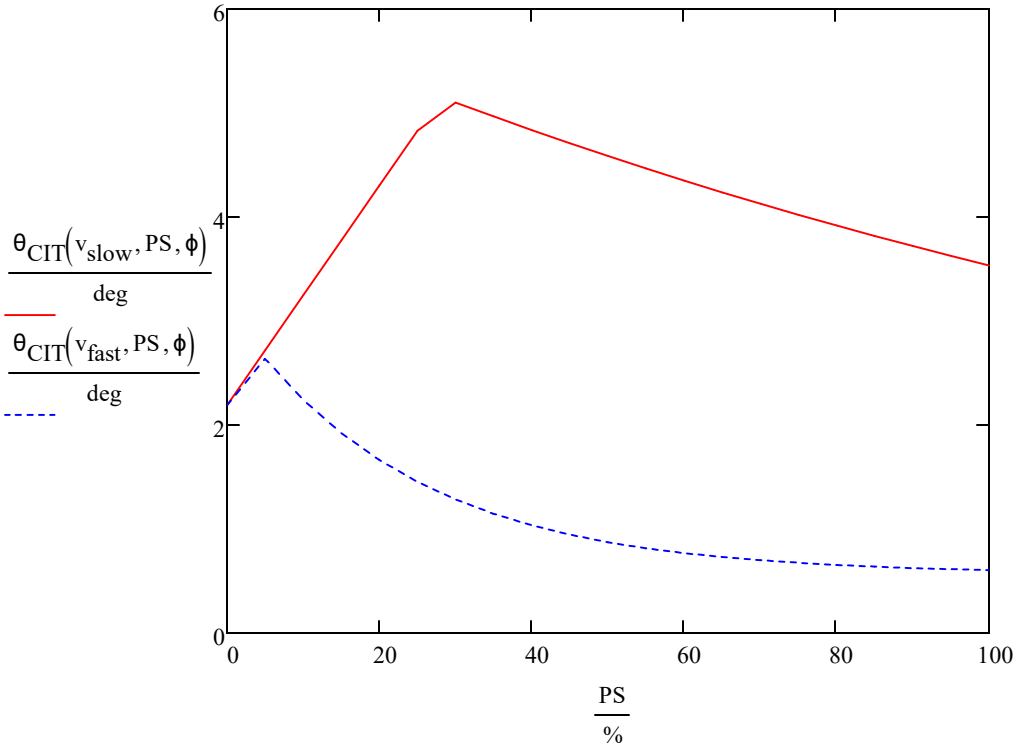
For a slow stun shot, where **cut-induced throw (CIT)** is maximum, CIT for a given percentage of inside spin (PS) is:

$$\theta_{CIT}(v, PS, \phi) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$$

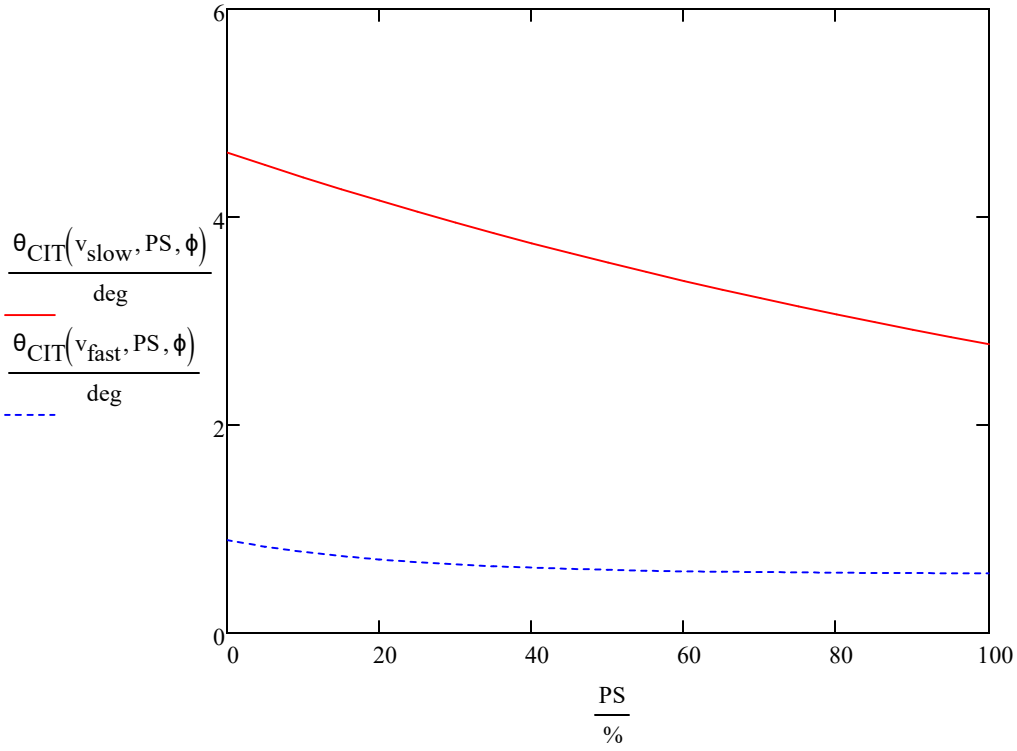
Here is how cut-induced throw (CIT) varies with percentage inside spin (PS) for slow and fast shots:

$PS := 0, .05 .. 1$

small cut angle: $\phi := 15 \cdot deg$



large cut angle: $\phi := 60 \cdot deg$



Now finding the percentage inside spin that results in the maximum cut-induced throw for a range of cut angles:

$\phi A := (0 \cdot deg \quad 15 \cdot deg \quad 30 \cdot deg \quad 45 \cdot deg \quad 60 \cdot deg \quad 75 \cdot deg \quad 85 \cdot deg)^T$ $PS := 0.95$

initial guess

$v := v_{slow}$

$i := 0$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

$i := 1$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

$i := 2$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

$i := 3$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

$i := 4$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

$i := 5$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

$i := 6$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

$PIS_{slow} := PIS$

$v := v_{fast}$

$i := 0$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

$i := 1$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

$i := 2$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

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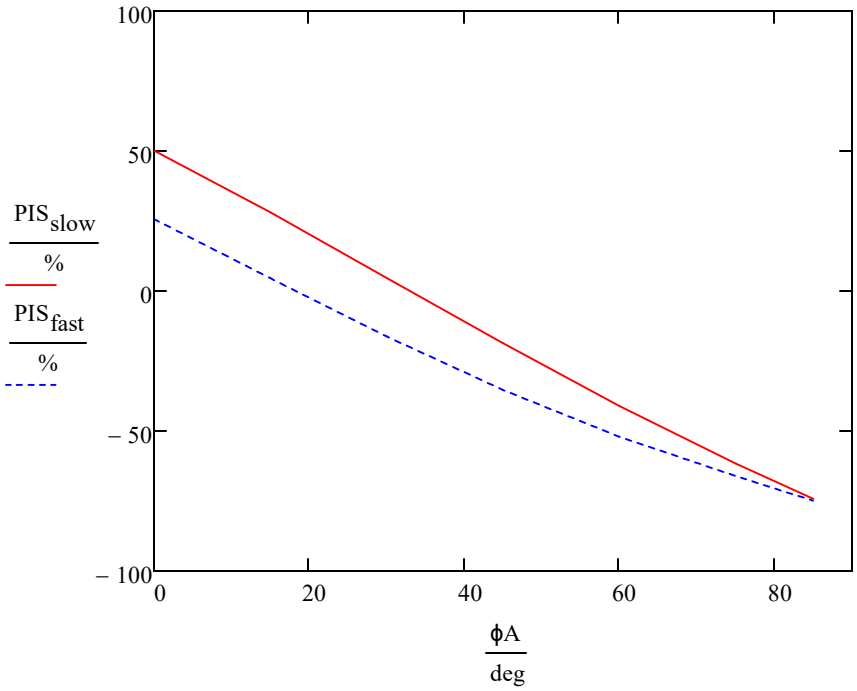
$i := 6$ $\phi := \phi A_i$ $\theta_{CIT}(PS) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$ $PIS_i := Maximize(\theta_{CIT}, PS)$

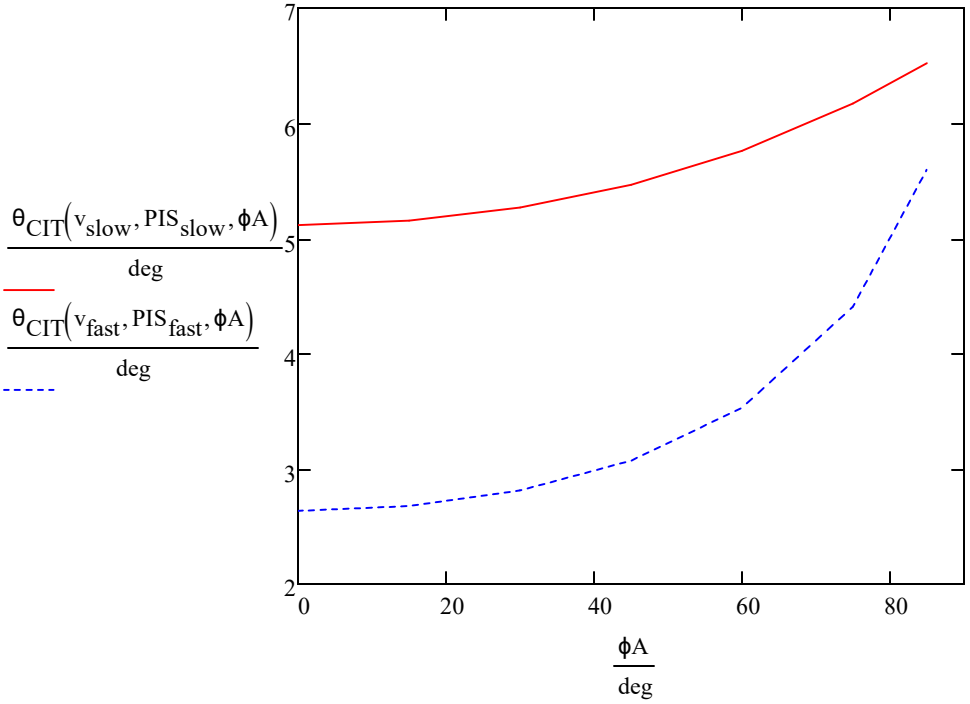
$PIS_{fast} := PIS$

Here is how the percentage of inside spin (PIS) required for maximum cut-induced throw (CIT) varies with cut angle (ϕA) for both slow and fast shots:



And here's how much throw you can get at each of those angles (for the given CB speed):

$$\theta_{CIT}(v, PS, \phi) := \theta_{throw}(v, 0, -\omega(v, PS), \phi)$$



With a very small cut-angle shot, only about 50% of maximum inside spin is required to get maximum cut-induced throw (CIT) at slow speed. For large cut angles, outside spin is required for maximum CIT (with larger amounts of outside spin required at larger angles), and more throw is possible (for a given CB speed).