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# TP B.30
# Outside vs. Inside Spin Margins for Error

supporting:
"The Illustrated Principles of Pool and Billiards"
**http://billiards.colostate.edu**


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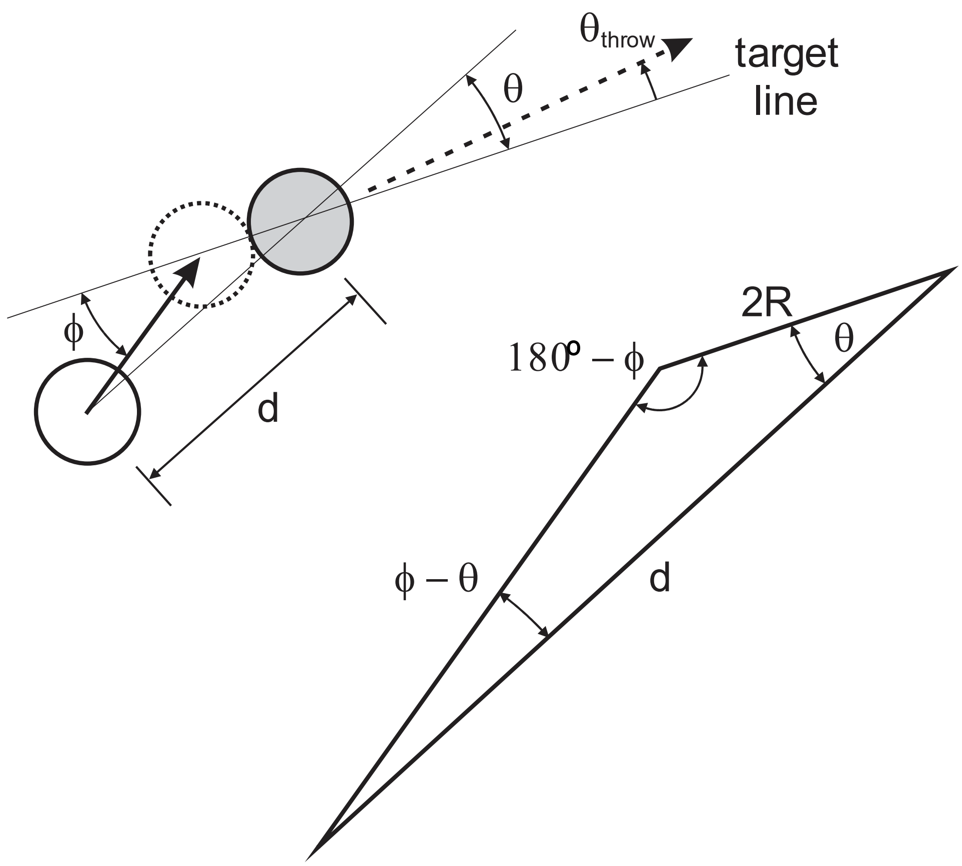



ball radius in m: $R := \frac{1.125 \cdot in}{m}$

shot distances in m: $d_{short} := \frac{1 \cdot ft}{m}$ $d_{long} := \frac{6 \cdot ft}{m}$

assumed CB angle error: $\Delta\varphi := 2 \cdot deg$

From the triangle above, using the Law of Sines:

$$\frac{\sin(180 \cdot deg - \varphi)}{d} = \frac{\sin(\varphi - \theta)}{2R}$$

Solving for the shot angle for a given shot distance d and cut angle φ gives:

$$\theta(\varphi, d) := \varphi - \operatorname{asin}\left(\frac{2R}{d} \cdot \sin(\varphi)\right)$$

From TP A.14, throw is calculated with the following, where speeds are in units of m/s:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} := \begin{pmatrix} 9.951 \times 10^{-3} \\ 0.108 \\ 1.088 \end{pmatrix} \qquad \mu(v) := a + b \cdot e^{-c \cdot v}$$

$$v_{rel}(v, \omega_x, \omega_z, \phi) := \sqrt{(v \cdot \sin(\phi) - R \cdot \omega_z)^2 + (R \cdot \omega_x \cdot \cos(\phi))^2}$$

$$\theta_{throw}(v, \omega_x, \omega_z, \phi) := \operatorname{atan}\left[\frac{\min\left(\frac{\mu(v_{rel}(v, \omega_x, \omega_z, \phi)) \cdot v \cdot \cos(\phi)}{v_{rel}(v, \omega_x, \omega_z, \phi)}, \frac{1}{7}\right) \cdot (v \cdot \sin(\phi) - R \cdot \omega_z)}{v \cdot \cos(\phi)}\right]$$

From **http://billiards.colostate.edu/faq/speed/typical/**, a typical range of shot speeds, converted to m/s is:

$$v_{slow} := \frac{1 \cdot mph}{\frac{m}{s}} = 0.447 \qquad v_{fast} := \frac{7 \cdot mph}{\frac{m}{s}} = 3.129$$

From TP A.25, percentage spin (PS) is related to spin rate ω (rad/sec) with:

$$\omega(v, PS) := \frac{5}{4} \cdot \frac{v}{R} \cdot PS$$

A slight error in CB direction ($\Delta\varphi$) can create a slightly full or slightly thin hit, relative to the ideal aim:

$$\varphi_{full} = \varphi - \frac{\Delta\varphi}{2} \qquad \varphi_{thin} = \varphi + \frac{\Delta\varphi}{2}$$

Taking throw into account, this will result in the following OB direction angles:

$$\theta_{thin}(\varphi, d, v, \omega_x, \omega_z) := \theta\left(\varphi + \frac{\Delta\varphi}{2}, d\right) + \theta_{throw}\left(v, \omega_x, \omega_z, \varphi + \frac{\Delta\varphi}{2}\right)$$

$$\theta_{full}(\varphi, d, v, \omega_x, \omega_z) := \theta\left(\varphi - \frac{\Delta\varphi}{2}, d\right) + \theta_{throw}\left(v, \omega_x, \omega_z, \varphi - \frac{\Delta\varphi}{2}\right)$$

And the total OB angle error ($\Delta\theta$) for a given CB angle error ($\Delta\varphi$) can be found with:

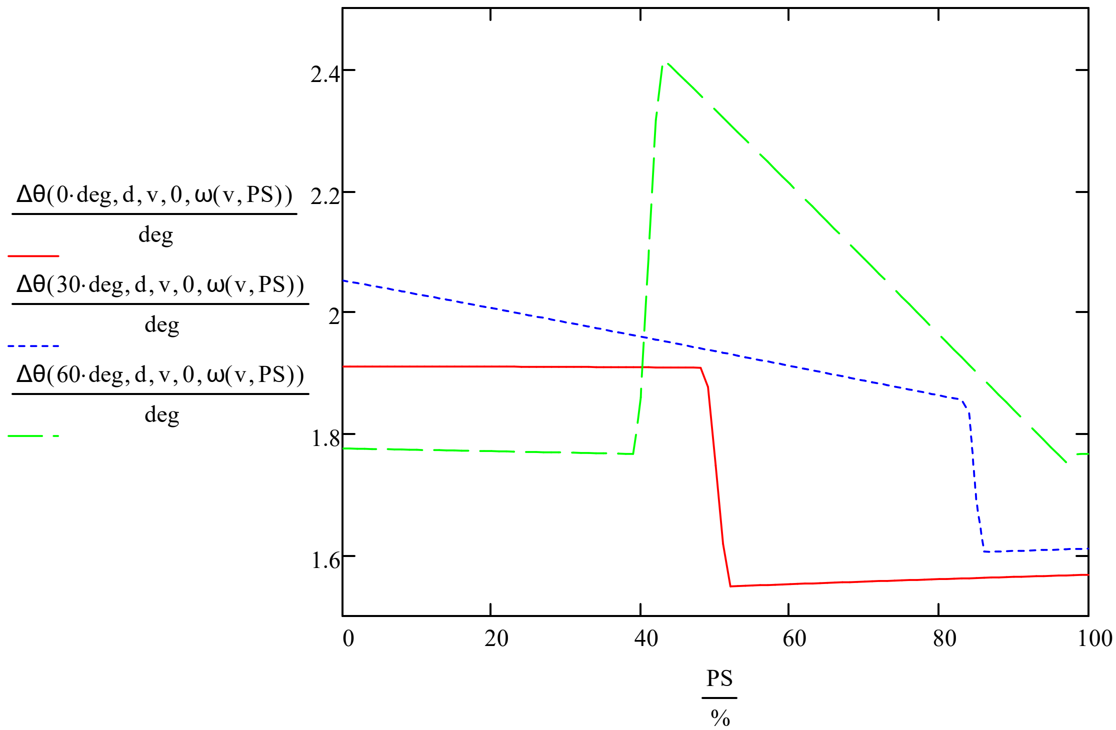
$$\Delta\theta(\varphi, d, v, \omega_x, \omega_z) := \theta_{thin}(\varphi, d, v, \omega_x, \omega_z) - \theta_{full}(\varphi, d, v, \omega_x, \omega_z)$$

**Now let's look at how OB angle error varies with cut angle and percentage spin for a given CB angle error:**

$$\Delta\varphi = 2\cdot deg$$

**short, slow, stun shot with different cut angles for different amount of outside spin:**

$d := d_{short}$ $v := v_{slow}$ $PS := 0\cdot\%, 1\cdot\% .. 100\%$

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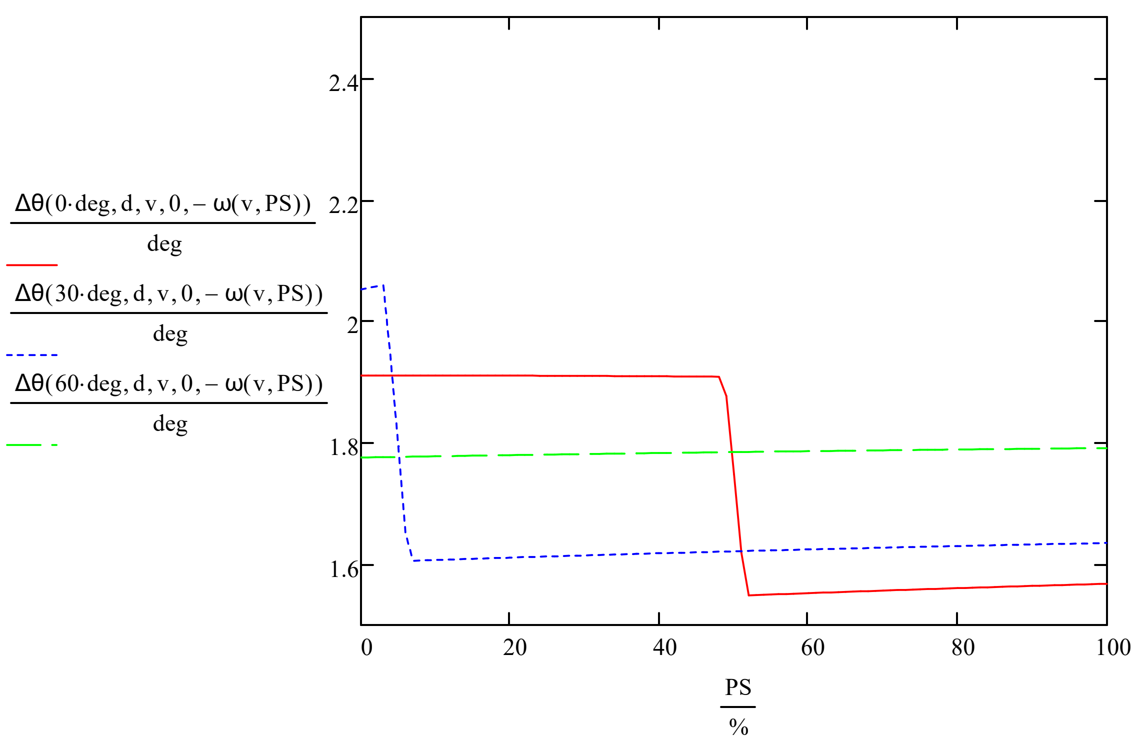
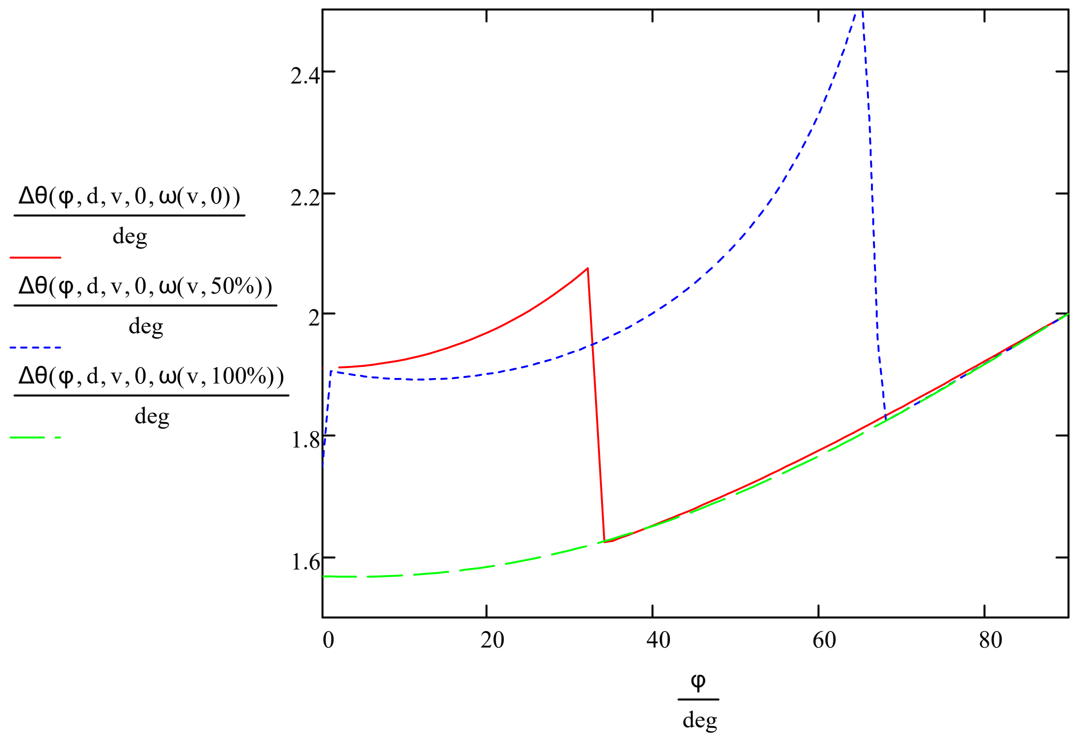


**short, slow, stun shot with different amount of outside spin for different cut angles:**

$d := d_{short}$ $\quad v := v_{slow}$ $\quad \varphi := 0 \cdot deg, 1 \cdot deg .. 90deg$

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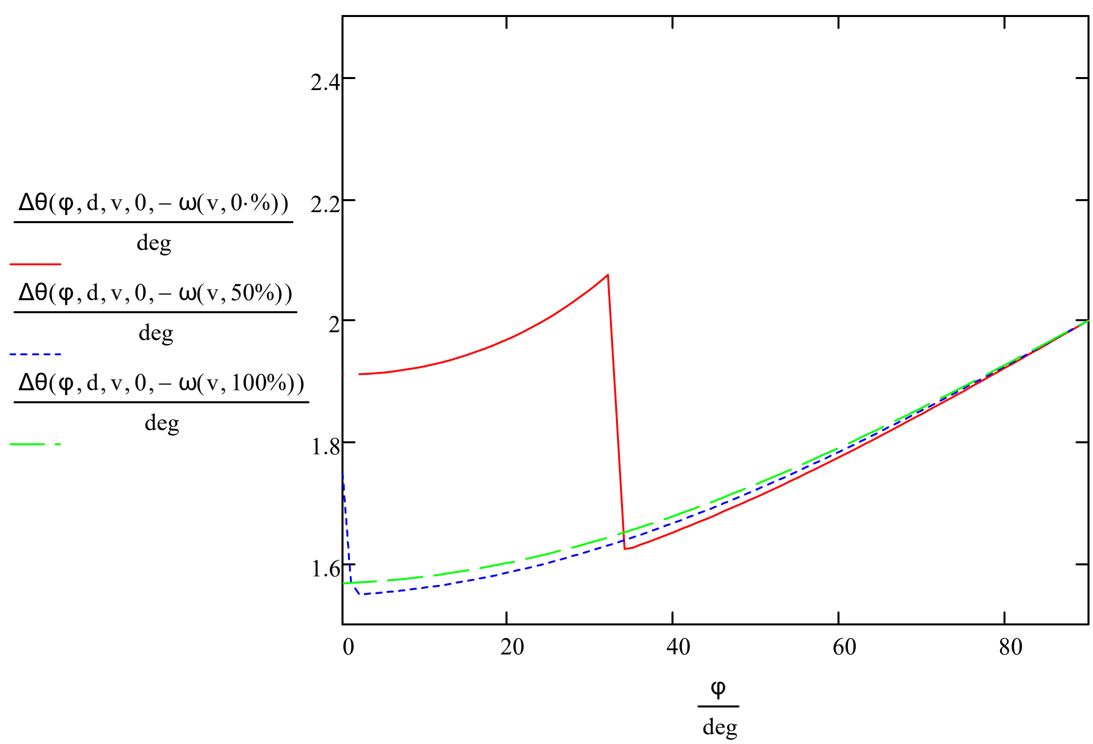
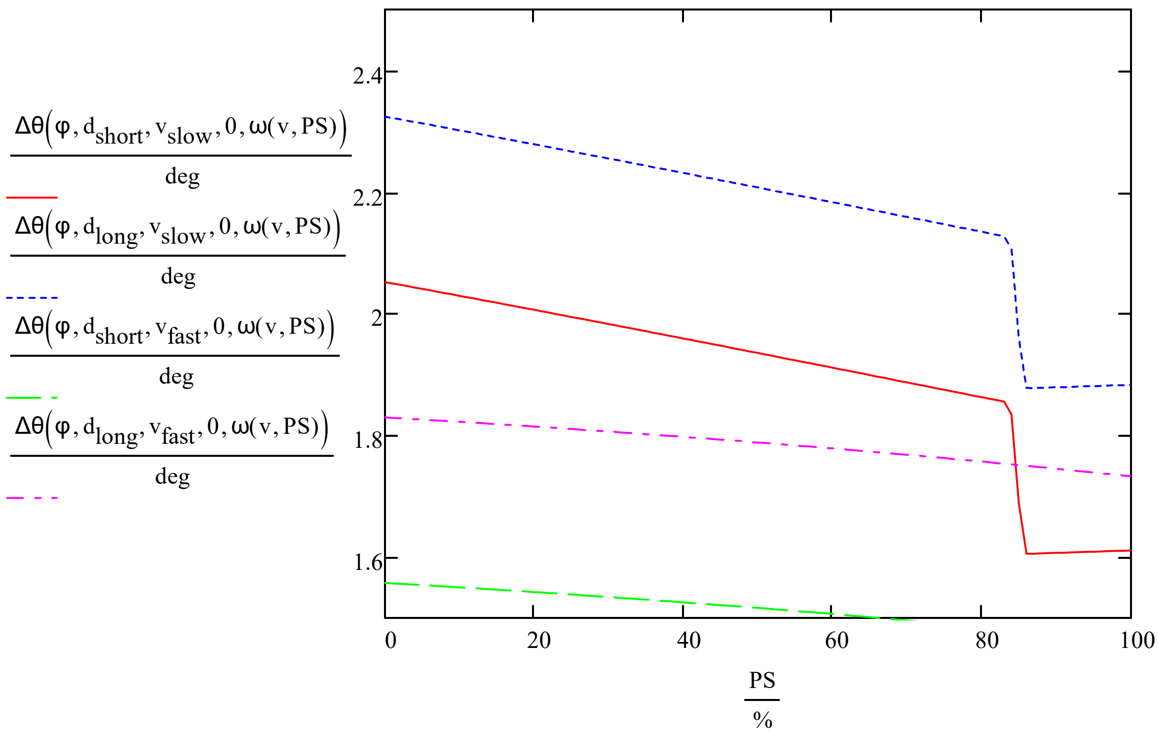


**slow, 1/2-ball-hit, shot with different shot distances and speeds for different amount of outside spin:**

$\varphi := 30 \cdot deg \qquad PS := 0 \cdot \%, 1 \cdot \% .. 100\%$

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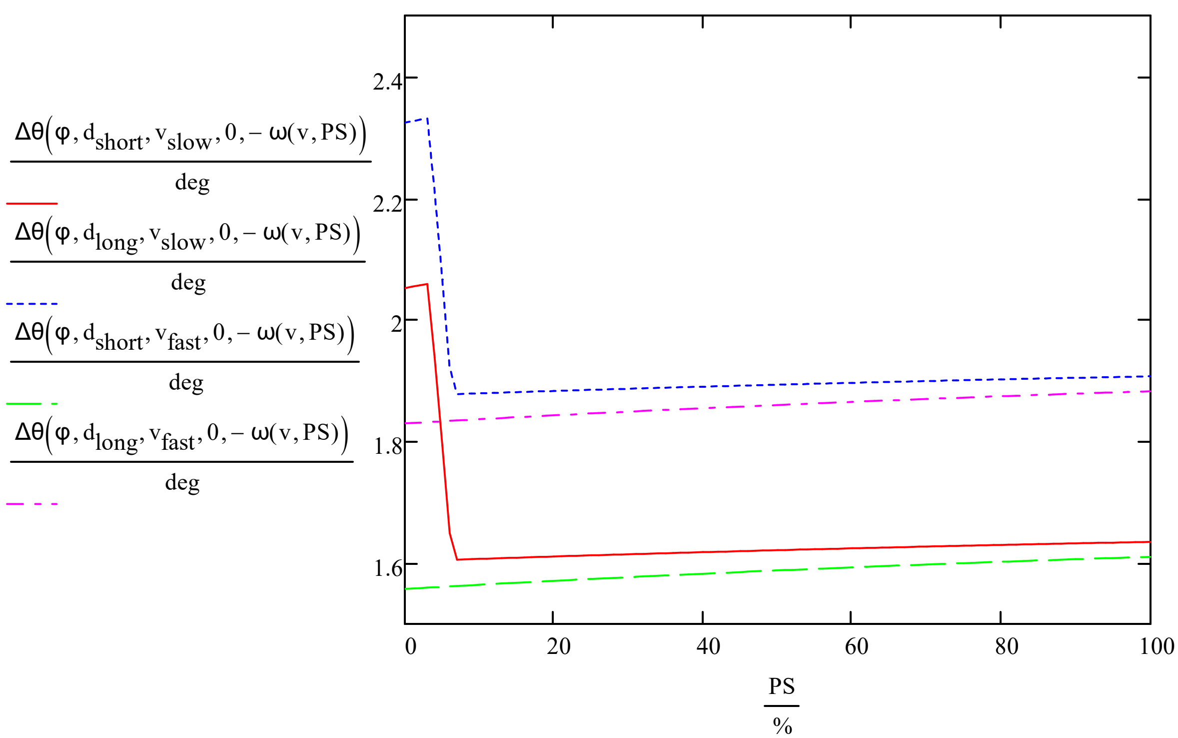

**Observations about and conclusions from the analysis:**

- For **background and advice** concerning the use of outside and inside spin, see these resource pages:
  **billiards.colostate.edu/faq/sidespin/outside-gearing**
  **billiards.colostate.edu/faq/sidespin/inside/**

- This analysis assumed that **CB deflection** is being corrected for based on shot speed and distance and on the amount of sidespin being used. Not everybody will be as good at this as others. For more info and advice, see:
  **billiards.colostate.edu/faq/sidespin/aim/compensation/**

- **For some (but not all) cut angles and some spin amounts, outside spin reduces OB angle error** (i.e., there is more room for error) than no spin, but this is certainly not true over a full range of angles and spins.

- **Inside spin** generally **reduces OB angle error** (i.e., there is more room for CB angle error) and makes it more consistent, as compared to outside spin.

- The amount of OB **angle error varies a lot** with the amount of spin when using **outside spin**. This is not the case with inside spin.

- **Faster speed** (assuming you can remain accurate with your stroke at faster speed) **reduces** the OB angle **error** (i.e., there is more room for CB angle error).

- OB angle **error** is obviously **larger** with **longer distance** shots (i.e., there is less room for CB angle error).