technical proof

TP B.25
Percentage Outside Spin Required for Maximum SIT at Any Cut Angle

supporting:
"The Illustrated Principles of Pool and Billiards"
https://drdavepoolinfo.com/
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From TP A.14, throw is calculated with the following, where speeds are in units of m/s:

$$R := \frac{1.125 \cdot in}{m} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} := \begin{pmatrix} 9.951 \times 10^{-3} \\ 0.108 \\ 1.088 \end{pmatrix} \qquad \mu(v) := a + b \cdot e^{-c \cdot v}$$

$$v_{rel}(v, \omega_x, \omega_z, \phi) := \sqrt{\left(v \cdot \sin(\phi) - R \cdot \omega_z\right)^2 + \left(R \cdot \omega_x \cdot \cos(\phi)\right)^2}$$

$$\theta_{throw}(v, \omega_x, \omega_z, \phi) := \operatorname{atan}\left[\frac{\min\left(\frac{\mu\left(v_{rel}(v, \omega_x, \omega_z, \phi)\right) \cdot v \cdot \cos(\phi)}{v_{rel}(v, \omega_x, \omega_z, \phi)}, \frac{1}{7}\right) \cdot \left(v \cdot \sin(\phi) - R \cdot \omega_z\right)}{v \cdot \cos(\phi)}\right]$$

From **https://drdavepoolinfo.com/faq/speed/typical/**, a typical range of CB speeds, converted to m/s is:

$$v_{slow} := \frac{1 \cdot mph}{\frac{m}{s}} = 0.447 \qquad v_{medium} := \frac{3 \cdot mph}{\frac{m}{s}} = 1.341 \qquad v_{fast} := \frac{7 \cdot mph}{\frac{m}{s}} = 3.129$$

From TP A.25, percentage spin (PS) is related to spin rate ω (rad/sec) with:

$$\omega(v, PS) := \frac{5}{4} \cdot \frac{v}{R} \cdot PS$$

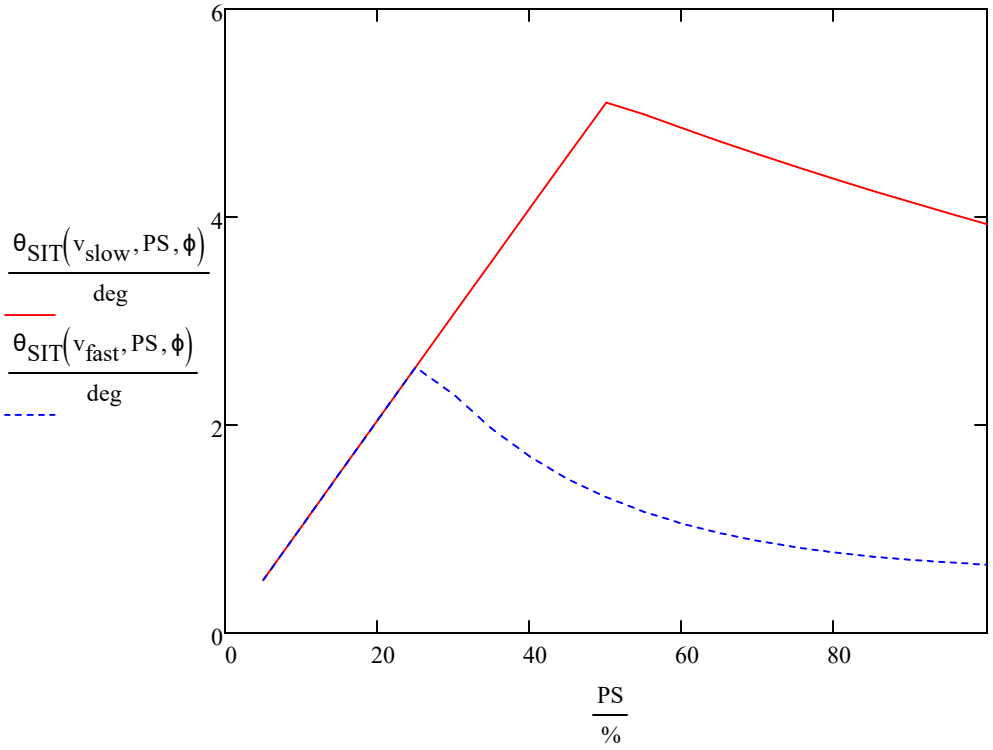
For a slow stun shot, where **spin-induced throw (SIT)** is maximum, SIT for a given percentage outside spin (PS) is:

$$\theta_{SIT}(v, PS, \phi) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$$

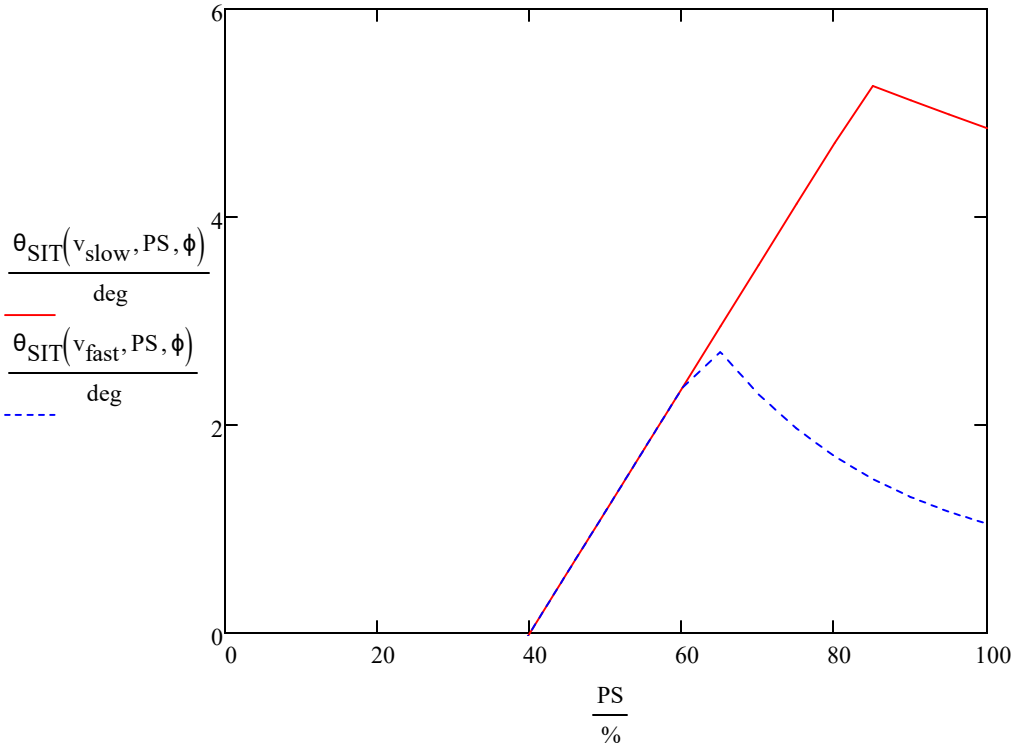
Here is how spin-induced throw (SIT) varies with percentage outside spin (PS) for slow and fast shots:

$PS := 0, .05 .. 1$

straight shot: $\phi := 0$



1/2-ball hit: $\phi := 30 \cdot deg$



Now finding the percentage outside spin that results in the maximum spin-induced throw for a range of cut angles:

$\phi A := (0 \cdot deg \quad 15 \cdot deg \quad 30 \cdot deg \quad 45 \cdot deg \quad 60 \cdot deg \quad 75 \cdot deg \quad 85 \cdot deg)^T$ $PS := 0.95$

initial guess

$v := v_{slow}$

$i := 0$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

$i := 1$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

$i := 2$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

$i := 3$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

$i := 4$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

$i := 5$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

$i := 6$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

$POS_{slow} := POS$

$v := v_{fast}$

$i := 0$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

$i := 1$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

$i := 2$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

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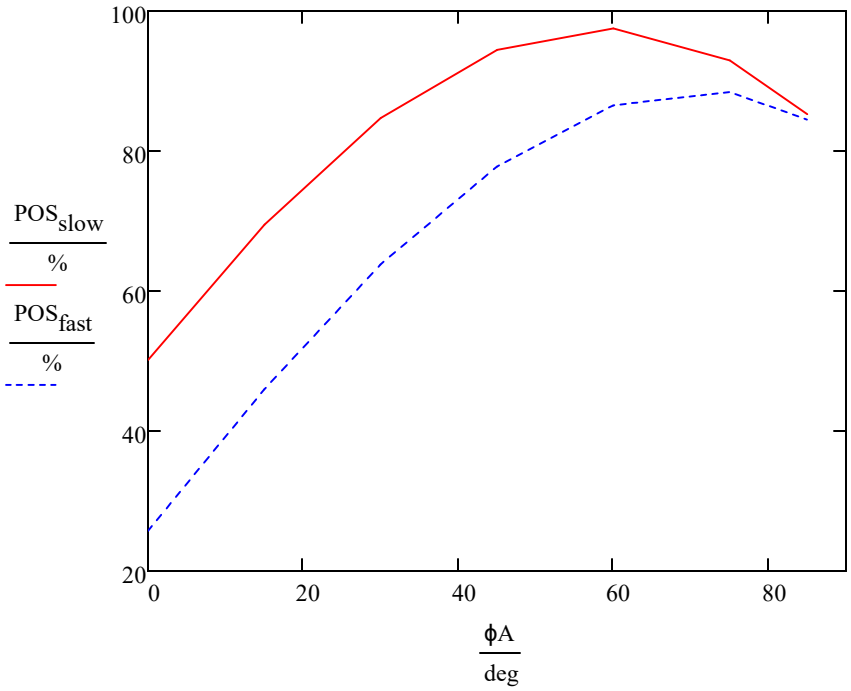
$i := 6$ $\phi := \phi A_i$ $\theta_{SIT}(PS) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$ $POS_i := Maximize(\theta_{SIT}, PS)$

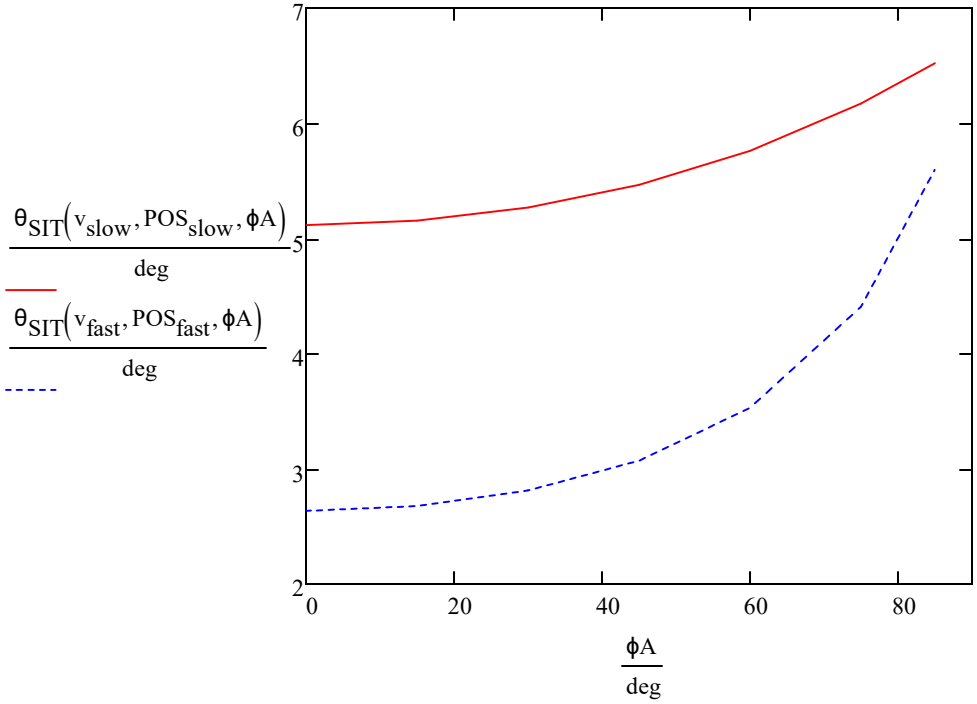
$POS_{fast} := POS$

Here is how the percentage outside spin (POS) required for maximum spin-induced throw (SIT) varies with cut angle (ϕA) for both slow and fast shots:



And here's how much throw you can get at each of those angles:

$$\theta_{SIT}(v, PS, \phi) := -\theta_{throw}(v, 0, \omega(v, PS), \phi)$$



With a fairly straight shot, only about 50% of maximum outside spin is required to get maximum spin-induced throw (SIT) at slow speed. For large cut angles, much more outside spin is required for maximum SIT, and more throw is possible (for a given CB speed).